Using the ordinal calculator

For ordCalc_0.2

Paul Budnik
Mountain Math Software
paul@mtnmath.com

Source code and documentation can be downloaded at
www.mtnmath.com/ord
and sourceforge.net/projects/ord.

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About this manual

This manual duplicates the online documentation. In this formatted version of the documentation, computer input and output is in `tty font`. For a complete description of the program and the mathematics on which it is based see the paper "A Computational Approach to the Ordinal Numbers". That document includes this manual as an appendix.

1 Introduction

The Ordinal Calculator is an interactive tool for understanding the hierarchies of recursive and countable ordinals[7, 6, 4]. It is also a research tool to help to expand these hierarchies. Its motivating goal is ultimately to expand the foundations of mathematics by using computer technology to manage the combinatorial explosion in complexity that comes with explicitly defining the recursive ordinals implicitly defined by the axioms of Zermelo-Frankel set theory[3, 1]. The underlying philosophy focuses on what formal systems tell us about physically realizable combinatorial processes[2].

 The ordinal calculator source code, documentation and some executables can be downloaded from: `http://www.mtnmath.com/ord` or `https://sourceforge.net/projects/ord`.

Most of this manual is automatically extracted from the online documentation.

This is a command line interactive interface to a program for exploring the ordinals. It supports recursive ordinals up to and beyond the the Bachmann-Howard ordinal[5]. It defines notations for the Church-Kleene ordinal and some larger countable ordinals. We refer to these as admissible level ordinals. They are used in a form of ordinal collapsing to define large recursive ordinals.

1.1 Command line options

The standard name for the ordinal calculator is `ord`. Typing `ord` (or `./ord`) ENTER will start `ord` in command line mode on most Unix or Linux based systems. The other command line options are mostly for validating or documenting `ord`. They are:

‘`cmd`’ — Read specified command file and enter command line mode.
‘`version`’ — Print program version.
‘`help`’ — Describe command line options.
‘`cmdDoc`’ — Write manual for command line mode in TeX format.
‘`tex`’ — Output TeX documentation files.
‘`psi`’ — Do tests of Veblen hierarchy.
‘`base`’ — Do tests of base class Ordinal.
‘`try`’ — Do tests of class FiniteFuncOrdinal.
‘`iter`’ — Do tests of class IterFuncOrdinal.
‘`admis`’ — Do tests of class AdmisLevOrdinal.
‘`admis2`’ — Do additional tests of class AdmisLevOrdinal.
‘`play`’ — Do integrating tests.
‘`descend`’ — Test descending trees.
‘`collapse`’ — Ordinal collapsing tests.
‘`nested`’ — Ordinal nested collapsing tests.
‘`nested2`’ — Ordinal nested collapsing tests 2.
‘`nested3`’ — Ordinal nested collapsing tests 3.
‘`exitCode`’ — LimitElement exit code base test.
‘`exitCode2`’ — LimitElement exit code base test 2.
‘`limitEltExitCode`’ — Admissible level LimitElement exit code test 0.
‘`limitEltExitCode1`’ — Admissible level limitElement exit code test 1.
‘`limitEltExitCode2`’ — Admissible level limitElement exit code test 2.
‘`limitEltExitCode3`’ — Admissible level limitElement exit code test 3.
‘`limitOrdExitCode`’ — Admissible level limitOrd exit code test.
‘`limitOrdExitCode1`’ — Admissible level limitOrd exit code test.
‘`limitOrdExitCode2`’ — Admissible level limitOrd exit code test.
‘`limitOrdExitCode3`’ — Admissible level limitOrd exit code test.
‘`transition`’ — Admissible level transition test.
‘`cmpExitCode`’ — Admissible level compare exit code test.
‘`drillDownExitCode`’ — Admissible level compare exit code test.
‘`embedExitCode`’ — Admissible level compare exit code test.
‘`fixedPoint`’ — test fixed point detection.
‘`helpTex`’ — TeX document command line options.

1.2 Help topics

Following are topics you can get more information about by entering ‘`help` topic’.

‘`cmds`’ – lists commands.

'`defined`' – list predefined ordinal variables.
'`compare`' – describes comparison operators.
'`members`' – describes member functions.
'`ordinal`' – describes available ordinal notations.
'`ordlist`' – describes ordinal lists and their use.
'`purpose`' – describes the purpose and philosophy of this project.
'`syntax`' – describes syntax.
'`version`' – displays program version.

This program supports GNU 'readline' input line editing.
You can download the program and documentation at: Mountain Math Software or at SourceForge.net (`http://www.mtnmath.com/ord` or `https://sourceforge.net/projects/ord`).

2 Ordinals

Ordinals are displayed in TeX and plain text format. (Enter 'help opts' to control this.) The finite ordinals are the nonnegative integers. The ordinal operators are +, * and ^ for addition, multiplication and exponentiation. Exponentiation has the highest precedence. Parenthesis can be used to group subexpressions.

The ordinal of the integers, `omega`, is represented by the single lowercase letter: '`w`'. The Veblen function is specified as '`psi(p1,p2,...,pn)`' where n is any integer > 0. Special notations are displayed in some cases. Specifically `psi(x)` is displayed as `w^x`. `psi(1,x)` is displayed as `epsilon(x)`. `psi(1,x,0)` is displayed as `gamma(x)`. In all cases the displayed version can be used as input.

Larger ordinals are specified as `psi_{px}(p1,p2,...,pn)`. The first parameter is enclosed in brackets not parenthesis. `psi_{1}` is defined as the union of `w, epsilon(0), gamma(0), psi(1, 0, 0, 0), psi(1, 0, 0, 0, 0), psi(1, 0, 0, 0, 0, 0), psi(1, 0, 0, 0, 0, 0, 0),` ... You can access the sequence whose union is a specific ordinal using member functions. Type `help members` to learn more about this. Larger notations beyond the recursive ordinals are also available in this implementation. See the documentation 'A Computational Approach to the Ordinal Numbers' to learn about 'Countable admissible ordinals'.

There are several predefined ordinals. '`w`' and '`omega`' can be be used interchangeably for the ordinal of the integers and in other contexts. '`eps0`' and '`omega1CK`' are also predefined. Type 'help defined' to learn more.

3 Predefined ordinals

The predefined ordinal variables are:
`omega` $= \omega$
`w` $= \omega$

`omega1CK` $= \omega_1$
`w1` $= \omega_1$
`w1CK` $= \omega_1$
`eps0` $= \varepsilon_0$

4 Syntax

The syntax is that of restricted arithmetic expressions and assignment statements. The tokens are variable names, nonnegative integers and the operators: +, * and ^ (addition, multiplication and exponentiation). Comparison operators are also supported. Type '`help comparison`' to learn about them. The letter 'w' is predefined as omega, the ordinal of the integers. Type '`help defined`' for a list of all predefined variables. To learn more about ordinals type '`help ordinal`'. C++ style member functions are supported with a '.' separating the variable name (or expression enclosed in parenthesis) from the member function name. Enter '`help members`' for the list of member functions.

An assignment statement or ordinal expression can be entered and it will be evaluated and displayed in normal form. Typing '`help opts`' lists the display options. Assignment statements are stored. They can be listed (command 'list') and their value can be used in subsequent expressions. All statements end at the end of a line unless the last character is '\'. Lines can be continued indefinitely. Comments must be preceded by either '`%`' or '`//`'.

Commands can be entered as one or more names separated by white space. File names should be enclosed in double quotes (") if they contain any non alphanumeric characters such as dot, '.'. Command names can be used as variables. Enter '`help cmds`' to get a list of commands and their functions.

5 Ordinal lists

Lists are a sequence of ordinals. An assignment statement can name a single ordinal or a list of them separated by commas. In most circumstances only the first element in the list is used, but some functions (such as member function 'limitOrdLst') use the full list. Type 'help members' to learn more about 'limitOrdLst'.

6 Commands

6.1 All commands

The following commands are available:
'`cmpCheck`' – toggle comparison checking for debugging.
'`examples`' – shows examples.
'`exit`' – exits the program.

'exportTeX' – exports assignments statements in TeX format.
'help' – displays information on various topics.
'list' – lists assignment statements.
'log' – writes a log file (ord.log default).
'listTeX' – lists assignment statements in TeX format.
'logopt' – controls the log file.
'opts' – controls display format and other options.
'prompt' – prompts for ENTER with optional string argument.
'quit' – exits the program.
'read' – read "input file" (ord_calc.ord default).
'readall' – same as read but no 'wait for ENTER' prompt.
'save' – saves assignment statements to a file (ord_calc.ord default).
'setDbg' – set debugging options.
'yydebug' – enables parser debugging (off option).

6.2 Commands with options

Following are the commands with options.

Command 'examples' – shows examples.
It has one parameter with the following options.
'arith' – demonstrates ordinal arithmetic.
'compare' – shows compare examples.
'display' – shows how display options work.
'member' – demonstrates member functions.
'VeblenFinite' – demonstrates Veblen functions of a finite number of ordinals.
'VeblenExtend' – demonstrates Veblen functions iterated up to a recursive ordinal.
'admissible' – demonstrates countable admissible level ordinal notations.
'admissibleDrillDown' – demonstrates admissible notations dropping down one level.
'admissibleContext' – demonstrates admissible ordinal context parameters.
'list' – shows how lists work.
'desLimitOrdLst' – shows how to construct a list of descending trees.

Command 'logopt' – controls the log file.
It has one parameter with the following options.
'flush' – flush log file.
'stop' – stop logging.

Command 'opts' – controls display format and other options.
It has one parameter with the following options.
'both' – display ordinals in both plain text and TeX formats.
'tex' – display ordinals in TeX format only.

'text' – display ordinals in plain text format only (default).
'psi' – additionally display ordinals in Psi format (turned off by the above options).
'promptLimit' – lines to display before pausing, <4 disables pause.

Command 'setDbg' – set debugging options.
It has one parameter with the following options.
'all' – turn on all debugging.
'arith' – debug ordinal arithmetic.
'clear' – turn off all debugging.
'compare' – debug compare.
'exp' – debug exponential.
'limArith' – limited debugging of arithmetic.
'limit' – debug limit element functions.
'construct' – debug constructors.

7 Member functions

Every ordinal (except 0) is the union of smaller ordinals. Every limit ordinal is the union of an infinite sequence of smaller ordinals. Member functions allow access to to these smaller ordinals. One can specify how many elements of this sequence to display or get the value of a specific instance of the sequence. For a limit ordinal, the sequence displayed, were it extended to infinity and its union taken, that union would equal the original ordinal.

The syntax for a member function begins with either an ordinal name (from an assignment statement) or an ordinal expression enclosed in parenthesis. This is followed by a dot (.) and then the member function name and its parameters enclosed in parenthesis. The format is 'ordinal_name.memberFunction(p)' where p may be optional. Functions 'limitOrdLst' and 'desLimitOrdLst' return a list all other member functions return a scalar value. Unless specified otherwise, the returned value is that of the ordinal the function was called from.

The member functions are:

'descend' – (n,m) iteratively (up to m) take nth limit element.
'descendFull' – (n,m,k) iteratively (up to m) take n limit elements with root k.
'getCompareIx' – display admissible compare index.
'limitElt' – evaluates to specified finite limit element.
'limitElement' – an alias for 'limitElt'.
'listLimitElts' – lists specified (default 10) limit elements.
'listElts' – alias for listLimitElts.
'limitOrd' – evaluates to specified (may be infinite) limit element.
'limitType' – return limitType.
'limitOrdLst' – apply each input from list to limitOrd and return that list.
'desLimitOrdLst' – (depth, list) does limitOrdLst iteratively on all outputs depth times.

‘`maxLimitType`’ – return maxLimitType.
‘`maxParameter`’ – return maxParameter (for debugging).

8 Comparison operators

Any two ordinals or ordinal expressions can be compared using the operators: <, <=, >, >= and ==. The result of the comparison is the text either TRUE or FALSE. Comparison operators have lower precedence than ordinal operators.

9 Examples

In the examples a line that begins with the standard prompt ‘ordCalc> ’ contains user input. All other lines contain program output

To select an examples type ‘`examples`’ followed by one of the following options.
‘`arith`’ – demonstrates ordinal arithmetic.
‘`compare`’ – shows compare examples.
‘`display`’ – shows how display options work.
‘`member`’ – demonstrates member functions.
‘`VeblenFinite`’ – demonstrates Veblen functions of a finite number of ordinals.
‘`VeblenExtend`’ – demonstrates Veblen functions iterated up to a recursive ordinal.
‘`admissible`’ – demonstrates countable admissible level ordinal notations.
‘`admissibleDrillDown`’ – demonstrates admissible notations dropping down one level.
‘`admissibleContext`’ – demonstrates admissible ordinal context parameters.
‘`list`’ – shows how lists work.
‘`desLimitOrdLst`’ – shows how to construct a list of descending trees.

9.1 Simple ordinal arithmetic

The following demonstrates ordinal arithmetic.

```
ordCalc> a=w^w
Assigning ( w^w ) to ‘a’.
ordCalc> b=w*w
Assigning ( w^2 ) to ‘b’.
ordCalc> c=a+b
Assigning ( w^w ) + ( w^2 ) to ‘c’.
ordCalc> d=b+a
Assigning ( w^w ) to ‘d’.
```

9.2 Comparison operators

The following shows compare examples.

```
ordCalc> psi(1,0,0) == gamma(0)
TRUE
ordCalc> psi(1,w) == epsilon(w)
TRUE
ordCalc> w^w < psi(1)
FALSE
ordCalc> psi(1)
Normal form:  w
```

9.3 Display options

The following shows how display options work.

```
ordCalc> a=w^(w^w)
Assigning ( w^( w^w ) ) to ‘a’.
ordCalc> b=epsilon(a)
Assigning epsilon( ( w^( w^w ) )) to ‘b’.
ordCalc> c=gamma(b)
Assigning gamma( epsilon( ( w^( w^w ) )) ) to ‘c’.
ordCalc> list
a = ( w^( w^w ) )
b = epsilon( ( w^( w^w ) ))
c = gamma( epsilon( ( w^( w^w ) )) )
ordCalc> opts tex
ordCalc> list
a = \omega{}^{\omega{}^{\omega{}}}
b = \varepsilon_{\omega{}^{\omega{}^{\omega{}}}}
c = \varphi( \varepsilon_{\omega{}^{\omega{}^{\omega{}}}}, 0, 0)
ordCalc> opts both
ordCalc> list
a = ( w^( w^w ) )
a = \omega{}^{\omega{}^{\omega{}}}
b = epsilon( ( w^( w^w ) ))
b = \varepsilon_{\omega{}^{\omega{}^{\omega{}}}}
c = gamma( epsilon( ( w^( w^w ) )) )
c = \varphi( \varepsilon_{\omega{}^{\omega{}^{\omega{}}}}, 0, 0)
```

9.4 Member functions

The following demonstrates member functions.

```
ordCalc> a=psi(1,0,0,0,0)
Assigning psi( 1, 0, 0, 0, 0 ) to ‘a’.
ordCalc> a.listElts(3)

3 limitElements for psi( 1, 0, 0, 0, 0 )
le(1) = psi( 1, 0, 0, 0 )
le(2) = psi( psi( 1, 0, 0, 0 ) + 1, 0, 0, 0 )
le(3) = psi( psi( psi( 1, 0, 0, 0 ) + 1, 0, 0, 0 ) + 1, 0, 0, 0 )
End limitElements

Normal form:  psi( 1, 0, 0, 0, 0 )
ordCalc> b=a.limitElt(6)
Assigning psi( psi( psi( psi( psi( psi( 1, 0, 0, 0 ) + 1, 0, 0, 0 ) + 1, 0, 0,
0 ) + 1, 0, 0, 0 ) + 1, 0, 0, 0 ) + 1, 0, 0, 0 ) to ‘b’.
```

9.5 Veblen function of N ordinals

The following demonstrates Veblen functions of a finite number of ordinals.

The Veblen function with a finite number of parameters, psi(x1,x2,...xn) is built up from the function omega^x. psi(x) = omega^x. psi(1,x) enumerates the fixed points of omega^x. This is epsilon(x). Each additional variable diagonalizes the functions definable with existing variables. These functions can have any finite number of parameters.

```
ordCalc> a=psi(w,w)
Assigning psi( w, w ) to ‘a’.
ordCalc> b=psi(a,3,1)
Assigning psi( psi( w, w ), 3, 1 ) to ‘b’.
ordCalc> b.listElts(3)

3 limitElements for psi( psi( w, w ), 3, 1 )
le(1) = psi( psi( w, w ), 3, 0 ) + 1
le(2) = psi( psi( w, w ), 2, psi( psi( w, w ), 3, 0 ) + 1 )
le(3) = psi( psi( w, w ), 2, psi( psi( w, w ), 2, psi( psi( w, w ), 3, 0 ) + 1
) + 1 )
End limitElements

Normal form:  psi( psi( w, w ), 3, 1 )
ordCalc> c=psi(a,a,b,1,3)
Assigning psi( psi( w, w ), psi( w, w ), psi( psi( w, w ), 3, 1 ), 1, 3 ) to ‘c’.
ordCalc> c.listElts(3)

3 limitElements for psi( psi( w, w ), psi( w, w ), psi( psi( w, w ), 3, 1 ), 1,
3 )
```

```
le(1) = psi( psi( w, w ), psi( w, w ), psi( psi( w, w ), 3, 1 ), 1, 2 ) + 1
le(2) = psi( psi( w, w ), psi( w, w ), psi( psi( w, w ), 3, 1 ), 0, psi( psi(
w, w ), psi( w, w ), psi( psi( w, w ), 3, 1 ), 1, 2 ) + 1 )
le(3) = psi( psi( w, w ), psi( w, w ), psi( psi( w, w ), 3, 1 ), 0, psi( psi(
w, w ), psi( w, w ), psi( psi( w, w ), 3, 1 ), 0, psi( psi( w, w ), psi( w, w
), psi( psi( w, w ), 3, 1 ), 1, 2 ) + 1 ) + 1 )
End limitElements

Normal form:  psi( psi( w, w ), psi( w, w ), psi( psi( w, w ), 3, 1 ), 1, 3 )
```

9.6 Extended Veblen function

The following demonstrates Veblen functions iterated up to a recursive ordinal.

The extended Veblen function, psi_{a}(x1,x2,...,xn), iterates the idea of the Veblen function up to any recursive ordinal. The first parameter is the recursive ordinal of this iteration.

```
ordCalc> a=psi_{1}(1)
Assigning psi_{ 1}(1) to 'a'.
ordCalc> a.listElts(4)

4 limitElements for psi_{ 1}(1)
le(1) = ( w^( psi_{ 1} + 1 ) )
le(2) = psi( psi_{ 1} + 1, 0 )
le(3) = gamma( psi_{ 1} + 1 )
le(4) = psi( psi_{ 1} + 1, 0, 0, 0 )
End limitElements

Normal form:  psi_{ 1}(1)
ordCalc> b=psi_{w+1}(3)
Assigning psi_{ w + 1}(3) to 'b'.
ordCalc> b.listElts(4)

4 limitElements for psi_{ w + 1}(3)
le(1) = psi_{ w}(psi_{ w + 1}(2) + 1)
le(2) = psi_{ w}(psi_{ w + 1}(2) + 1, 0)
le(3) = psi_{ w}(psi_{ w + 1}(2) + 1, 0, 0)
le(4) = psi_{ w}(psi_{ w + 1}(2) + 1, 0, 0, 0)
End limitElements

Normal form:  psi_{ w + 1}(3)
```

9.7 Admissible countable ordinal notations

The following demonstrates countable admissible level ordinal notations.

Countable admissible level notations, omega_{k,g}(x1,x3,..,xn) extend the idea of recursive notation to larger countable ordinals. The first parameter is the admissible level. omega_{1} is the Church-Kleene ordinal. The remaining parameters are similar to those defined for smaller ordinal notations.

```
ordCalc> a=w_{1}(1)
Assigning omega_{ 1}(1) to 'a'.
ordCalc> a.listElts(4)

4 limitElements for omega_{ 1}(1)
le(1) = psi_{ omega_{ 1} + 1}
le(2) = psi_{ psi_{ omega_{ 1} + 1} + 1}
le(3) = psi_{ psi_{ psi_{ omega_{ 1} + 1} + 1} + 1}
le(4) = psi_{ psi_{ psi_{ psi_{ omega_{ 1} + 1} + 1} + 1} + 1}
End limitElements

Normal form:  omega_{ 1}(1)
```

9.8 Admissible notations drop down parameter

The following demonstrates admissible notations dropping down one level.

Admissible level ordinals have the a limit sequence defined in terms of lower levels. The lowest admissible level is that of recursive ordinals. To implement this definition of limit sequence, a trailing parameter in square brackets is used. This parameter (if present) defines an ordinal at one admissible level lower than indicated by other parameters.

```
ordCalc> a=w_{1}[1]
Assigning omega_{ 1}[ 1] to 'a'.
ordCalc> a.listElts(4)

4 limitElements for omega_{ 1}[ 1]
le(1) = w
le(2) = psi_{ w}
le(3) = psi_{ psi_{ w} + 1}
le(4) = psi_{ psi_{ psi_{ w} + 1} + 1}
End limitElements

Normal form:  omega_{ 1}[ 1]
ordCalc> b=w_{1}
Assigning omega_{ 1} to 'b'.
```

```
ordCalc> c=b.limitOrd(w^3)
Assigning omega_{ 1}[ ( w^3 )] to ‘c’.
ordCalc> c.listElts(4)

4 limitElements for omega_{ 1}[ ( w^3 )]
le(1) = omega_{ 1}[ ( w^2 )]
le(2) = omega_{ 1}[ (( w^2 )*2 )]
le(3) = omega_{ 1}[ (( w^2 )*3 )]
le(4) = omega_{ 1}[ (( w^2 )*4 )]
End limitElements

Normal form:  omega_{ 1}[ ( w^3 )]
ordCalc> d=w_{5,c}(3,0)
Assigning omega_{ 5, omega_{ 1}[ ( w^3 )]}(3, 0) to ‘d’.
ordCalc> d.listElts(4)

4 limitElements for omega_{ 5, omega_{ 1}[ ( w^3 )]}(3, 0)
le(1) = omega_{ 5, omega_{ 1}[ ( w^3 )]}(2, 1)
le(2) = omega_{ 5, omega_{ 1}[ ( w^3 )]}(2, omega_{ 5, omega_{ 1}[ ( w^3 )]}(2,
1) + 1)
le(3) = omega_{ 5, omega_{ 1}[ ( w^3 )]}(2, omega_{ 5, omega_{ 1}[ ( w^3 )]}(2,
omega_{ 5, omega_{ 1}[ ( w^3 )]}(2, 1) + 1) + 1)
le(4) = omega_{ 5, omega_{ 1}[ ( w^3 )]}(2, omega_{ 5, omega_{ 1}[ ( w^3 )]}(2,
omega_{ 5, omega_{ 1}[ ( w^3 )]}(2, omega_{ 5, omega_{ 1}[ ( w^3 )]}(2, 1) + 1)
+ 1) + 1)
End limitElements

Normal form:  omega_{ 5, omega_{ 1}[ ( w^3 )]}(3, 0)
```

9.9 Admissible notations context parameter

The following demonstrates admissible ordinal context parameters.

The context parameter in admissible level ordinals allows one to use any notation at any admissible level to define notations at any lower admissible level or to define recursive ordinals.

```
ordCalc> a=[[1]]w_{1}
Assigning [[1]]omega_{ 1} to ‘a’.
ordCalc> a.listElts(4)

4 limitElements for [[1]]omega_{ 1}
le(1) = omega_{ 1}[ w]
le(2) = omega_{ 1}[ omega_{ 1}[ w]]
le(3) = omega_{ 1}[ omega_{ 1}[ omega_{ 1}[ w]]]
```

```
le(4) = omega_{ 1}[ omega_{ 1}[ omega_{ 1}[ omega_{ 1}[ w]]]]
End limitElements

Normal form:  [[1]]omega_{ 1}
```

9.10 Lists of ordinals

The following shows how lists work.

Lists are a sequence of ordinals (including integers). A list can be assigned to a variable just as a single ordinal can be. In most circumstances lists are evaluated as the first ordinal in the list. In 'limitOrdLst' all of the list entries are used. These member functions return a list with an input list

```
ordCalc> lst = 1, 12, w, gamma(w^w), w1
Assigning 1, 12, w, gamma( ( w^w ) ), omega_{ 1} to 'lst'.
ordCalc> a=w1.limitOrdLst(lst)
( omega_{ 1} ).limitOrd( 12 ) = omega_{ 1}[ 12]
( omega_{ 1} ).limitOrd( w ) = omega_{ 1}[ w]
( omega_{ 1} ).limitOrd( gamma( ( w^w ) ) ) = omega_{ 1}[ gamma( ( w^w ) )]
Assigning omega_{ 1}[ 12], omega_{ 1}[ w], omega_{ 1}[ gamma( ( w^w ) )] to 'a'.
ordCalc> bg = w_{w+33}
Assigning omega_{ w + 33} to 'bg'.
ordCalc> c=bg.limitOrdLst(lst)
( omega_{ w + 33} ).limitOrd( 12 ) = omega_{ w + 33}[ 12]
( omega_{ w + 33} ).limitOrd( w ) = omega_{ w + 33}[ w]
( omega_{ w + 33} ).limitOrd( gamma( ( w^w ) ) ) = omega_{ w + 33}[ gamma( ( w^w
) )]
( omega_{ w + 33} ).limitOrd( omega_{ 1} ) = omega_{ w + 33}[ omega_{ 1}]
Assigning omega_{ w + 33}[ 12], omega_{ w + 33}[ w], omega_{ w + 33}[ gamma( (
w^w ) )], omega_{ w + 33}[ omega_{ 1}] to 'c'.
```

9.11 List of descending trees

The following shows how to construct a list of descending trees.

'desLimitOrdLst' iterates 'limitOrdLst' to a specified 'depth'. The first parameter is the integer depth of iteration and the second is the list of parameters to be used. This routine first takes'limitOrd' of each element in the second parameter creating a list of outputs. It then takes this list and evaluates 'limitOrd' for each of these values at each entry in the original parameter list. All of these results are combined in a new list and the process is iterated 'depth' times. The number of results grows exponentially with 'depth'.

```
ordCalc> lst = 1, 5, w, psi(2,3)
```

```
Assigning 1, 5, w, psi( 2, 3 ) to 'lst'.
ordCalc> bg = w_{3}
Assigning omega_{ 3} to 'bg'.
ordCalc> d= bg.desLimitOrdLst(2,lst)
( omega_{ 3} ).limitOrd( 1 ) = omega_{ 3}[ 1]
( omega_{ 3} ).limitOrd( 5 ) = omega_{ 3}[ 5]
( omega_{ 3} ).limitOrd( w ) = omega_{ 3}[ w]
( omega_{ 3} ).limitOrd( psi( 2, 3 ) ) = omega_{ 3}[ psi( 2, 3 )]
Descending to 1 for omega_{ 3}
( omega_{ 3}[ 1] ).limitOrd( 1 ) = omega_{ 2}
( omega_{ 3}[ 1] ).limitOrd( 5 ) = omega_{ 2, omega_{ 2, omega_{ 2, omega_{ 2,
omega_{ 2} + 1} + 1} + 1} + 1}
( omega_{ 3}[ 5] ).limitOrd( 1 ) = omega_{ 3}[ 4]
( omega_{ 3}[ 5] ).limitOrd( 5 ) = omega_{ 2, omega_{ 2, omega_{ 2, omega_{ 2,
omega_{ 3}[ 4] + 1} + 1} + 1} + 1}
( omega_{ 3}[ w] ).limitOrd( 1 ) = omega_{ 3}[ 1]
( omega_{ 3}[ w] ).limitOrd( 5 ) = omega_{ 3}[ 5]
( omega_{ 3}[ psi( 2, 3 )] ).limitOrd( 1 ) = omega_{ 3}[ psi( 2, 2 ) + 1]
( omega_{ 3}[ psi( 2, 3 )] ).limitOrd( 5 ) = omega_{ 3}[ epsilon( epsilon( epsilon(
epsilon( psi( 2, 2 ) + 1) + 1) + 1) + 1)]
Assigning omega_{ 3}[ 1], omega_{ 3}[ 5], omega_{ 3}[ w], omega_{ 3}[ psi( 2, 3
)], omega_{ 2}, omega_{ 2, omega_{ 2, omega_{ 2, omega_{ 2, omega_{ 2} + 1} + 1}
+ 1} + 1}, omega_{ 3}[ 4], omega_{ 2, omega_{ 2, omega_{ 2, omega_{ 2, omega_{ 3}[
4] + 1} + 1} + 1} + 1}, omega_{ 3}[ 1], omega_{ 3}[ 5], omega_{ 3}[ psi( 2, 2 )
+ 1], omega_{ 3}[ epsilon( epsilon( epsilon( epsilon( psi( 2, 2 ) + 1) + 1) +
1) + 1)] to 'd'.
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Index

The defining reference for a phrase, if it exists, has the page *number* in *italics*.

This index is semiautomated with multiple entries created for some phrases and subitems automatically detected. Hand editing would improve things, but is not practical for a manual describing software and underlying theory both of which are evolving.

Symbols

A

B

C

D

E

T

V

W

Y

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